LAMINATES

1 Laminates

Laminates are plates, which are made by stacking a number of layers, also called plies or lamina's. Each ply can be fabricated from various materials, having different mechanical properties within a wide range. The properties may be isotropic, completely anisotropic, or orthotropic. The latter occurs when a ply is a composite material, consisting of a matrix which is enforced by long fibers in one direction. The material directions are denoted as 1 (fiber, longitudinal) and 2 (transversal).

To increase the bending stiffness of the laminate, the thickness can be enlarged, by applying a layer of filler material. e.g. a foam, which is considered to be isotropic. The figure shows an exploded view of a laminate with fiber reinforced plies. Also a foam laminate is shown.

The mechanical properties of the laminate are determined by its stiffness matrix, which can be calculated from the properties of the plies, their thicknesses and their stacking sequence. The behavior of the laminate is based on linear plate bending theory.

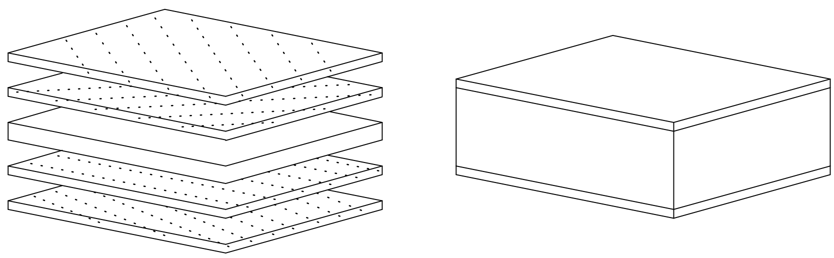

Fig. 1.1 : *Laminate as a stack of plies*

1.1 Ply strains

For one ply (k) the strain components in the global coordinate system can be related to the strain in the mid-plane and the curvature of the mid-plane. The strain components in the material coordinate system – indicated with superscript $*$ – are calculated with the transformation matrix $\underline{T}_{\varepsilon}$.

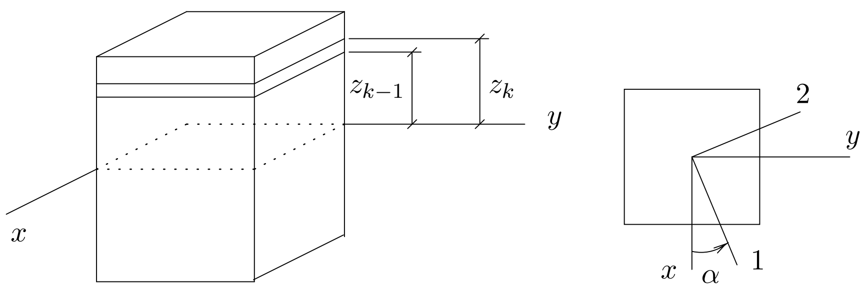

Fig. 1.2 : *A ply in a laminate*

$$\underset{\sim}{\varepsilon}_k(z) = \underset{\sim}{\varepsilon}_0 - z\underset{\sim}{\kappa} \quad \rightarrow \quad \underset{\sim}{\varepsilon}_k^*(z) = \underline{T}_{\varepsilon,k}\underset{\sim}{\varepsilon}_k(z)$$

with

$$\underline{T}_\varepsilon = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix}$$

1.2 Ply stresses

Assuming linearly elastic material behavior according to Hooke's law, the stress components in the material directions can be determined, using the material stiffness matrix $\underline{C}$. Stress components in the global coordinate system are calculated with the transformation matrix $\underline{T}_\sigma^{-1}$.

$$\underset{\sim}{\sigma}_k^* = \underline{C}_k^*\underset{\sim}{\varepsilon}_k^* \quad \rightarrow \quad \underset{\sim}{\sigma}_k = \underline{T}_{\sigma,k}^{-1}\underline{C}_k^*\underline{T}_{\varepsilon,k}\underset{\sim}{\varepsilon}_k = \underline{C}_k\underset{\sim}{\varepsilon}_k = \underline{C}_k(\underset{\sim}{\varepsilon}_0 - z\underset{\sim}{\kappa})$$

with

$$\underline{T}_\sigma = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix} \quad ; \quad \underline{T}_\sigma^{-1} = \begin{bmatrix} c^2 & s^2 & -2cs \\ s^2 & c^2 & 2cs \\ cs & -cs & c^2 - s^2 \end{bmatrix}$$

1.3 ABD-matrix

The cross-sectional forces and moments can be determined by summation of the integrated stress components over each individual ply. The result is the so-called ABD-matrix, which relates cross-sectional forces and moments to mid-plane strains and curvatures.

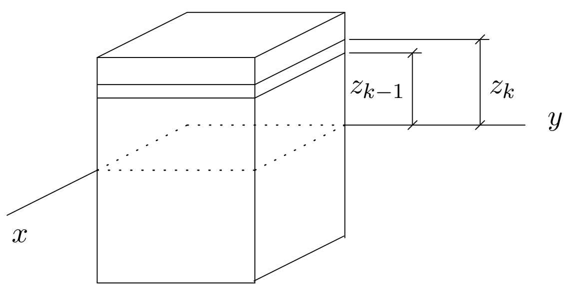

Fig. 1.3 : *A ply in a laminate*

$$
\begin{aligned}
\underset{\sim}{N}_k &= \int_{z_{k-1}}^{z_k} \underset{\sim}{\sigma}_k \, dz = (z_k - z_{k-1})\underline{C}_k \underset{\sim}{\varepsilon}_0 - \tfrac{1}{2}(z_k^2 - z_{k-1}^2)\underline{C}_k \underset{\sim}{\kappa} = \underline{A}_k \underset{\sim}{\varepsilon}_0 + \underline{B}_k \underset{\sim}{\kappa} \\
\underset{\sim}{M}_k &= -\int_{z_{k-1}}^{z_k} \underset{\sim}{\sigma}_k \, z \, dz = -\tfrac{1}{2}(z_k^2 - z_{k-1}^2)\underline{C}_k \underset{\sim}{\varepsilon}_0 + \tfrac{1}{3}(z_k^3 - z_{k-1}^3)\underline{C}_k \underset{\sim}{\kappa} = \underline{B}_k \underset{\sim}{\varepsilon}_0 + \underline{D}_k \underset{\sim}{\kappa}
\end{aligned}
$$

summation over all plies

$$
\underset{\sim}{N} = \sum_{k=1}^{n} \underset{\sim}{N}_k = \underline{A}\,\underset{\sim}{\varepsilon}_0 + \underline{B}\,\underset{\sim}{\kappa} \qquad ; \qquad \underset{\sim}{M} = \sum_{k=1}^{n} \underset{\sim}{M}_k = \underline{B}\underset{\sim}{\varepsilon}_0 + \underline{D}\underset{\sim}{\kappa} \quad \rightarrow
$$

$$
\begin{bmatrix} \underset{\sim}{N} \\ \underset{\sim}{M} \end{bmatrix} = \begin{bmatrix} \underline{A} & \underline{B} \\ \underline{B} & \underline{D} \end{bmatrix} \begin{bmatrix} \underset{\sim}{\varepsilon}_0 \\ \underset{\sim}{\kappa} \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} \underset{\sim}{\varepsilon}_0 \\ \underset{\sim}{\kappa} \end{bmatrix} = \begin{bmatrix} \underline{a} & \underline{b} \\ \underline{b} & \underline{d} \end{bmatrix} \begin{bmatrix} \underset{\sim}{N} \\ \underset{\sim}{M} \end{bmatrix}
$$

The ABD-matrix characterizes the mechanical behavior of the laminate, as it relates the cross-sectional loads to strains and curvatures of the mid-plane. The figure shows the influence of some components of the ABD-matrix.

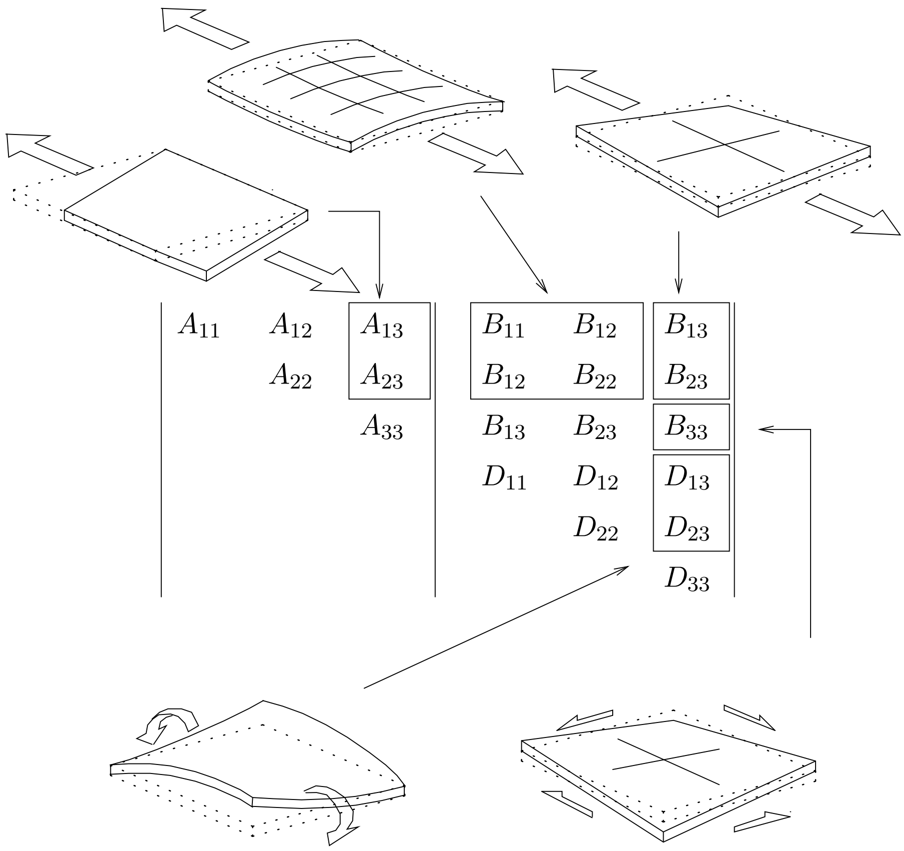

Fig. 1.4 : *ABD-matrix components and their associated deformations*

1.4 Thermal and humidity loading

When the laminate is subjected to a change in temperature, thermal strains will occur due to thermal expansion. In each ply these strains will generally be different, leading to deformation. Analogously, the laminate can be placed in a humid environment, which also causes strains caused by absorption.

The thermal and humidity strains can be calculated in each ply, given its coefficients of thermal expansion (α_1 and α_2) and humid driven expansion (β_1 and β_2). To calculate stresses these strains $\hat{\underset{\sim}{\varepsilon}}_k$ must be subtracted from the mechanical strains $\underset{\sim}{\varepsilon}_k^*$.

The cross-sectional forces and moments are related to the mid-plane strains and curvatures by the ABD-matrix. The forces caused by thermal and humidity expansion are simply added.

strains in ply k due to temperature change ΔT_k and absorption c_k

$$\begin{bmatrix} \hat{\varepsilon}_{11} \\ \hat{\varepsilon}_{22} \\ \hat{\gamma}_{12} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix} \Delta T_k + \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix} c_k \qquad \rightarrow \qquad \hat{\underset{\sim}{\varepsilon}}_k^* = \underset{\sim}{\alpha}_k \Delta T_k + \underset{\sim}{\beta}_k c_k$$

stresses due to thermal(absorption) and mechanical strains $\underset{\sim}{\varepsilon}_k^*$

$$\underset{\sim}{\sigma}_k^* = \underline{C}_k^* (\underset{\sim}{\varepsilon}_k^* - \hat{\underset{\sim}{\varepsilon}}_k^*) \quad \rightarrow \quad \underset{\sim}{\sigma}_k = \underline{C}_k (\underset{\sim}{\varepsilon}_k - \hat{\underset{\sim}{\varepsilon}}_k) = \underline{C}_k (\underset{\sim}{\varepsilon}_0 - z\underset{\sim}{\kappa} - \hat{\underset{\sim}{\varepsilon}}_k)$$

forces and moments in ply k :

$$\underset{\sim}{N}_k = \int_{z_{k-1}}^{z_k} \underset{\sim}{\sigma}_k \, dz = \underline{A}_k \underset{\sim}{\varepsilon}_0 + \underline{B}_k \underset{\sim}{\kappa} - \underline{A}_k \hat{\underset{\sim}{\varepsilon}}_k \quad \rightarrow \quad \underset{\sim}{N} = \sum_{k=1}^{n} \underset{\sim}{N}_k$$

$$\underset{\sim}{M}_k = -\int_{z_{k-1}}^{z_k} \underset{\sim}{\sigma}_k \, z \, dz = \underline{B}_k \underset{\sim}{\varepsilon}_0 + \underline{D}_k \underset{\sim}{\kappa} - \underline{B}_k \hat{\underset{\sim}{\varepsilon}}_k \quad \rightarrow \quad \underset{\sim}{M} = \sum_{k=1}^{n} \underset{\sim}{M}_k$$

$$\begin{bmatrix} \underset{\sim}{N} \\ \underset{\sim}{M} \end{bmatrix} = \begin{bmatrix} \underline{A} & \underline{B} \\ \underline{B} & \underline{D} \end{bmatrix} \begin{bmatrix} \underset{\sim}{\varepsilon}_0 \\ \underset{\sim}{\kappa} \end{bmatrix} - \begin{bmatrix} \hat{\underset{\sim}{N}} \\ \hat{\underset{\sim}{M}} \end{bmatrix} \quad \rightarrow$$

$$\begin{bmatrix} \underset{\sim}{\varepsilon}_0 \\ \underset{\sim}{\kappa} \end{bmatrix} = \begin{bmatrix} \underline{a} & \underline{b} \\ \underline{b} & \underline{d} \end{bmatrix} \left\{ \begin{bmatrix} \underset{\sim}{N} \\ \underset{\sim}{M} \end{bmatrix} + \begin{bmatrix} \hat{\underset{\sim}{N}} \\ \hat{\underset{\sim}{M}} \end{bmatrix} \right\}$$

1.5 Stacking

The fibers in the plies can be oriented randomly, however, mostly a certain orientation relative to the orientation in other plies is chosen, resulting in *cross-ply*, *angle-ply* and *regular angle-ply* laminates. Fiber orientation in plies above and under the mid-plane can be *symmetric* and *anti-symmetric*. The resulting mechanical behavior is related to certain components in the ABD-matrix.

The notation of the ply-orientations is illustrated with the next examples :

[0/0/0/90/90/90/45/..]	total stacking and orientation sequence is given
$[0_3/90_2/45/..]$	total stacking and orientation sequence given; index gives number of plies
$[0_3/90_2/45/-45_3]_s$	symmetric laminate

cross-ply	• orthotropic plies • material directions = global directions. • $A_{13} = A_{23} = 0$
angle-ply	• orthotropic plies • each ply material direction 1 is rotated over α^o w.r.t. global direction x.
regular angle-ply	• orthotropic plies • subsequent plies have material direction 1 rotated alternatingly over α^o and $-\alpha^o$ w.r.t. the global x-axis. • even number of plies $\rightarrow$ $A_{13} = A_{23} = 0$
symmetric	• symmetric stacking w.r.t. mid-plane • $\underline{B} = \underline{0}$
anti-symm.	• anti-symmetric stacking w.r.t. mid-plane • $D_{13} = D_{23} = 0$
quasi-isotropic	• $\alpha_k = k\frac{\pi}{n}$ with $k = 1, .., n$ (n = number of plies)

1.6 Recommendations for laminate stacking

There are no general rules for the number of plies in a laminate, their (mutual) fiber orientation and their stacking sequence. For most applications, however, there are some recommendations.

- Choose a symmetric laminate.
  Because $\underline{B} = \underline{0}$, there is no coupling between forces and curvatures and between strains and moments.
- Minimize material direction differences in subsequent plies.
  Large differences lead to high inter-laminar shear stresses, which may cause delamination.
- Minimize stiffness differences of subsequent plies.
  Large difference lead to high inter-laminar shear stresses.
- Avoid neighboring plies with the same orientation.
  The same orientation results in high inter-laminar shear stresses under thermal loading.
- Ensure negative inter-laminar normal stresses (pressure).
  Negative stresses (tensile) may lead to delamination.

1.7 Damage

Several damage phenomena can occur in a laminate :

- fibre rupture

- fibre buckling
- matrix cracking
- fibre-matrix de-adhesion
- interlaminar delamination

The occurence of these damage phenomena can be monitored with a damage criterion.

1.8 Damage : Tsai-Hill

For anisotropic materials the yield criterion of Tsai-Hill is often used. Either tensile (T_i) or compressive (C_i) yield limits are used. The shear limit is denoted as S. The yield criterion is used in each ply of the laminate in the material coordinate system.

Failure can also occur by exceeding the maximum strain in longitudinal (ε_{fl}) or transversal (ε_{ft}) direction.

Tsai-Hill yield criterion for tensile loading

$$\left(\frac{\sigma_{11}^2}{T_l^2}\right) - \left(\frac{\sigma_{11}\sigma_{22}}{T_l^2}\right) + \left(\frac{\sigma_{22}^2}{T_t^2}\right) + \left(\frac{\sigma_{12}^2}{S^2}\right) = 1$$

Tsai-Hill yield criterion for compressive loading

$$\left(\frac{\sigma_{11}^2}{C_l^2}\right) - \left(\frac{\sigma_{11}\sigma_{22}}{C_l^2}\right) + \left(\frac{\sigma_{22}^2}{C_t^2}\right) + \left(\frac{\sigma_{12}^2}{S^2}\right) = 1$$

1.9 Damage : ILSS

An important failure mode in laminates is the loss of adhesion between plies. This occurs when the interlaminar shear stress exceeds a limit value : the interlaminar shear strength (ILSS).

The interlaminar shear stresses (ils) are calculated as the difference between the global stress component values between two adjacent plies, indicated here as $(\ )_t$ (top) and $(\ )_b$ (bottom).

$$ils_{xx} = |\sigma_{xx_b} - \sigma_{xx_t}| \quad ; \quad ils_{yy} = |\sigma_{yy_b} - \sigma_{yy_t}| \quad ; \quad ils_{xy} = |\sigma_{xy_b} - \sigma_{xy_t}|$$

1.10 Material parameters for some materials

Material parameters for Carbon fibre, HP-PE fibre and epoxy are listed in the table[1]. Material parameters for fibre-matrix composites are also listed in the table. HP-PE fibres can be plasma treated (pl.tr.)to improve adhesion to the epoxy matrix The fibre volume fraction is $V_f = 0.5$.

fibre and matrix parameters						
	E_{11} [GPa]	E_{22} [GPa]	G_{12} [GPa]	G_{23} [GPa]	ν_{21} [-]	ν_{12} [-]
Carbon	230	20	20	8	0.013	0.23
HP-PE	80	2	0.8	0.8	0.010	0.30
Epoxy	3.4		1.2		0.37	

fibre/epoxy composite parameters $V_f = 0.5$					
		PVOH	HP-PE pl.tr.	HP-PE untr.	Aramid
E_l	[GPa]	22	49		65
E_t	[GPa]	4.2	3.2		4.8
G_{lt}	[GPa]	1.9	1.0		2.1
ν_{lt}	[-]	0.3	0.3		
T_l	[MPa]	690	1070	910	1150
T_t	[MPa]	10	8	2.5	14
C_l	[MPa]	140	91	73	240
C_t	[MPa]		44	21	
S	[MPa]	16	15	6	34
ε_{fl}	[%]	3.2	3.4		2.5
ε_{ft}	[%]	0.24	0.31		0.29
$ILSS$	[MPa]	>50	30		55

1.11 Examples

Some laminates have been modeled and analyzed with Matlab. For each example the laminate build-up and the loading is presented. The resulting stiffness matrix is shown. The loading results in strains and curvatures of the mid-plane. The deformation is visualized and the ply-strains and stresses are plotted, both in global directions and in fiber directions.

- Random 4-ply laminate
- Symmetric 8-ply laminate
- Anti-symmetric 8-ply laminate

[1] from PhD thesis T.Peijs

1.12 Random 4-ply laminate

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Laminate build-up (lam)
  z-      z+      ang     El      Et        nutl   Gl
  2.000   3.000   90.000 150.000  30.000    0.300  10.000
  1.000   2.000   45.000 100.000  25.000    0.200  20.000
  0.000   1.000    0.000 110.000  21.000    0.300  15.000
 -1.000   0.000   30.000  90.000  17.000    0.200  10.000
------------------------------------------------------------
Mechanical load {ld)
 [Nxx Nyy Nxy   Mxx Myy Mxy] = [ 100.00   0.00   0.00   100.00   0.00   0.00 ]
------------------------------------------------------------
Stiffness matrix
  2.57e+08   4.45e+07   4.15e+07    -1.83e+05  -3.98e+04  -1.71e+04
  4.45e+07   2.52e+08   2.83e+07    -3.98e+04  -4.62e+05  -2.37e+04
  4.15e+07   2.83e+07   7.55e+07    -1.71e+04  -2.37e+04  -6.53e+04

 -1.83e+05  -3.98e+04  -1.71e+04     3.77e+02   9.80e+01   5.17e+01
 -3.98e+04  -4.62e+05  -2.37e+04     9.80e+01   1.11e+03   4.73e+01
 -1.71e+04  -2.37e+04  -6.53e+04     5.17e+01   4.73e+01   1.43e+02
------------------------------------------------------------
Strains in the mid-plane (e0)
 [ exx  eyy  exy  ] = [ 3.810e-04 -1.079e-04  -3.102e-04 ]
Curvatures of the mid-plane (kr)
 [ kxx  kyy  kxy  ] = [ 4.784e-01 -6.892e-02  -2.637e-01 ]
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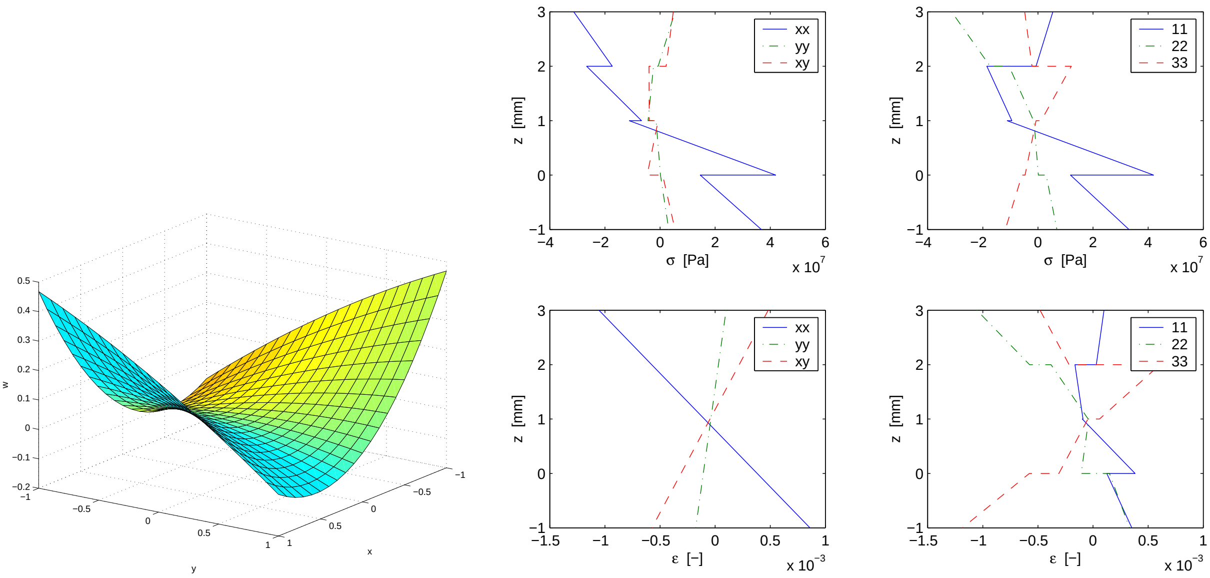

1.13 Symmetric 8-ply laminate

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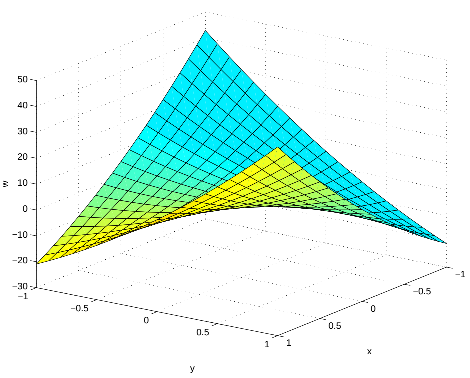
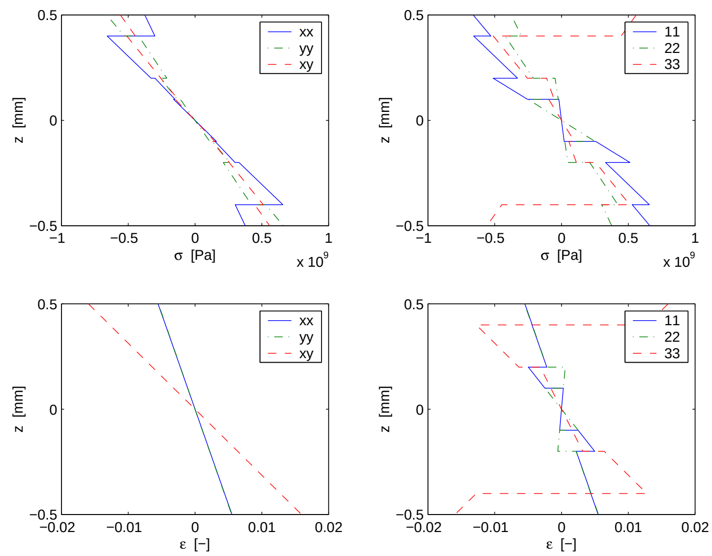
```
Laminate build-up (lam)
  z-       z+      ang      El       Et        nutl    Gl
  0.400    0.500   90.000 100.000   50.000    0.300   35.000
  0.200    0.400    0.000 120.000   70.000    0.300   40.000
  0.100    0.200   30.000 100.000   50.000    0.300   35.000
  0.000    0.100  -30.000 200.000  100.000    0.300   60.000
 -0.100   -0.000  -30.000 200.000  100.000    0.300   60.000
 -0.200   -0.100   30.000 100.000   50.000    0.300   35.000
 -0.400   -0.200    0.000 120.000   70.000    0.300   40.000
 -0.500   -0.400   90.000 100.000   50.000    0.300   35.000
------------------------------------------------------------
Mechanical load {ld)
 [Nxx Nyy Nxy   Mxx Myy Mxy] = [ 100.00   0.00 100.00   100.00 100.00 100.00 ]
------------------------------------------------------------
Stiffness matrix
  1.16e+08   2.13e+07  -2.82e+06    -2.12e-12   1.06e-13   1.25e-14
  2.13e+07   8.99e+07  -1.71e+06     1.06e-13  -8.47e-13  -5.71e-16
 -2.82e+06  -1.71e+06   4.19e+07     1.25e-14  -5.71e-16  -4.24e-13

 -2.12e-12   1.06e-13   1.25e-14     7.42e+00   1.55e+00   2.97e-02
  1.06e-13  -8.47e-13  -5.71e-16     1.55e+00   7.42e+00   4.58e-02
  1.25e-14  -5.71e-16  -4.24e-13     2.97e-02   4.58e-02   3.11e+00
------------------------------------------------------------
Strains in the mid-plane (e0)
 [ exx  eyy  exy  ] = [ 9.527e-07 -1.794e-07  2.444e-06 ]
Curvatures of the mid-plane (kr)
 [ kxx  kyy  kxy  ] = [ 1.106e+01 1.097e+01  3.190e+01 ]
============================================================
```





1.14 Anti-symmetric 8-ply laminate

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```
Laminate build-up (lam)
  z-       z+      ang      El       Et        nutl   Gl
  0.400    0.500   90.000 100.000  50.000    0.300   35.000
  0.200    0.400    0.000 120.000  70.000    0.300   40.000
  0.100    0.200   30.000 100.000  50.000    0.300   35.000
  0.000    0.100  -30.000 200.000 100.000    0.300   60.000
 -0.100   -0.000   30.000 200.000 100.000    0.300   60.000
 -0.200   -0.100  -30.000 100.000  50.000    0.300   35.000
 -0.400   -0.200   -0.000 120.000  70.000    0.300   40.000
 -0.500   -0.400  -90.000 100.000  50.000    0.300   35.000
------------------------------------------------------------
Mechanical load {ld)
 [Nxx Nyy Nxy   Mxx Myy Mxy] = [ 100.00   0.00 100.00   100.00 100.00 100.00 ]
------------------------------------------------------------
Stiffness matrix
  1.16e+08   2.13e+07   1.26e-11    -2.12e-12   1.06e-13  -5.45e+01
  2.13e+07   8.99e+07   1.00e-10     1.06e-13  -8.47e-13  -1.72e+02
  1.26e-11   1.00e-10   4.19e+07    -5.45e+01  -1.72e+02  -4.24e-13

 -2.12e-12   1.06e-13  -5.45e+01     7.42e+00   1.55e+00   4.82e-19
  1.06e-13  -8.47e-13  -1.72e+02     1.55e+00   7.42e+00   2.19e-18
 -5.45e+01  -1.72e+02  -4.24e-13     3.71e-18   2.19e-18   3.11e+00
------------------------------------------------------------
Strains in the mid-plane (e0)
 [ exx  eyy  exy  ] = [ 4.859e-06 6.048e-05  6.274e-05 ]
Curvatures of the mid-plane (kr)
 [ kxx  kyy  kxy  ] = [ 1.115e+01 1.115e+01  3.217e+01 ]
============================================================
```